

Approaches to the spreadsheets environment: the case of preservice teachers when they solve word problems

Wajeeh Daher

Abstract

Some previous researches studied the potentialities of the spreadsheets environment as an environment for solving word problems and the introduction of algebraic concepts like the variable, the unknown and the parameter. The paper discusses the work of preservice teachers in the spreadsheets environment to solve word problems, their use of the building potentialities and their utilization of the algebraic potentialities of the spreadsheets. It also discusses the role of the user in the various potentialities utilized.

Keywords: spreadsheets, spreadsheets potentialities, problem solving

1. Introduction

1.1 Spreadsheets as potential environment for the learning of mathematics:

Various researchers pointed at the potentials that spreadsheets bring to mathematics learning. Spreadsheets are suggested as technological tools to be integrated in the learning of various mathematical subjects (Abramovich 1995, Abramovich & Sugden 2004, Baker, Hvorecky & Sugden 2005, Neuwirth, 1995). Among the various mathematical subjects for which spreadsheets activities were suggested are: Algebra (Smith, 2005), Calculus (Abramovich & Sugden, 2004), Number theory (Abramovich, 1997), Combinatorics (Neuwirth, 1995), Numerical Analysis (Soper & Lee, 1994). For a more in-depth description of the research done on the integration of spreadsheets in mathematics education refer to Baker and Sugden (2003).

Some researchers were especially interested in the potential of spreadsheets to make an easy transition from Arithmetic to Algebra for the learner (Ainley, 1996, Dettori et al 1995, Felloy and Rojano 1989, Sutherland and Rojano 1993, Tall 1993). Ainley (1996) describes the spreadsheets as a tool

in an environment which situates the use of formal notation in meaningful contexts. Sutherland and Rojano (1993, p. 353) describe how the spreadsheets environment supported students to move from “thinking with the specific to the general both in terms of the unknown and the mathematical relationships expressed in the problem”. Dettori et al (1995) describe how students can learn the difference between variables and parameters using the spreadsheets.

1.2 Students solve word problems in the spreadsheets environment:

Sutherland and Rojano (1993) studied how 10-11 years old students work in the spreadsheets environment to represent and solve word problems in Algebra. Another goal of their research was to categorize the solution procedures along the dimension arithmetic-algebraic.

Ainley (1996) studied how the context of physical experiment and spreadsheets environment that utilizes the graphing potential of the spreadsheets can introduce 11 years old students to the power of generalizing through formal algebraic notation. This was done while the students were asked to solve word problems.

Abramovich and Nabors (1996, 1997) studied how middle school students solve word problems in the spreadsheets environment. Rojano (1996) studied how 15-16 years old students who had difficulties in mathematics solve word problems.

1.3 The role of the user:

The role of the user is becoming of increasing importance in the design of computer software and implementation of computer assisted learning materials (Hedberg and Perry, 1985; Ewusi-Mensah, 2003). Despite this increasing importance, very few researches (if any) have touched this issue regarding the spreadsheets environment.

1.4 Research questions:

In this research we were interested to explore the approaches that preservice teachers have towards the spreadsheets environment when they use it to solve word problems by themselves or as they expect primary school students to solve such problems. We wanted to explore these methods regarding the following issues: (1) the spreadsheets building potentialities (for example building columns, inserting functions, building controls) that the preservice teachers use when they solve word problems in the spreadsheets environment as themselves or as they expect primary school students to do so (2) the spreadsheets algebraic potentialities (for example generating unknowns, variable or parameters) that the preservice teachers utilize when they solve word problems in the spreadsheets environment as themselves or as they expect primary school students to do so (3) the role of the user that the preservice teachers or the primary school students draw for themselves, when coming to solve word problems in the spreadsheets environment (4) combining the solutions of the different parts of the word problem.

2. Research setting

2.1 Educational setting:

The experiments from which the observations in this research are described took place in two classes of second year preservice teachers who studied to graduate as teachers in mathematics and computer. The observations were held during the years: 2004-2006. The number of students was 27 in the year 2004-2005 and 34 in the year 2005-2006. These preservice teachers had already taken a course called “Spreadsheets Integration in Education”. I gave the preservice teachers four word problems taken from the literature. The word problems were modified to suit the preservice teachers who were required to solve them in the spreadsheets environment in two

ways: first as a 10-12 years old student in the primary school would solve them in the spreadsheets environment, second as they would do that if their solution would be different from the student's solution. The preservice teachers were also asked to find one method or frame through which all the parts of a word problem would be solved together. This assignment was given while discussing the role of spreadsheets in mathematics education in the course "didactics of mathematics teaching" which I gave them.

2.2 The word problem:

In this research I will report the various methods employed by the pre-service teachers to solve one of the word problems called 'The Measurement of a Field' problem, which was taken from Sutherland and Rojano (1993). The exact wording of the word problem follows:

- (a) The perimeter of a rectangular field measures 102 meters. The length of the field is twice as much as the width of the field. How much do the length and the width of the field measure?
- (b) What happens if the perimeter of the rectangular field measures 162 meters and the relation between the length and the width of the field stays the same?
- (c) What happens if the perimeter of the rectangular field measures 128 meters but the length of the field is three times its width?

2.3 Describing the role of the user:

To describe the role of the user we will depend on the work of Halliday (1985) that categorized the processes of the human in any text into six processes of which we are interested here in three processes:

- (1) Material processes which are processes of doing
- (2) Mental processes which are processes of sensing: feeling, thinking, seeing, etc
- (3) Behavioural processes which are of psychological and physiological nature, like thinking, worrying, dreaming.

3. Findings: The preservice teachers go to the spreadsheets

3.1 Solving the first part:

The preservice teachers solved the first part of the ‘Measurement of a Field’ problem, by themselves or as 10-12 years old students might solve it, in one of the following methods:

- (1) Solving as with pen and paper, i.e. not taking into account the spreadsheets potentialities: solving arithmetically. Figure 2 describes this method:

	A	B	C	D	E
1					
2	width	length	perimeter		
3	1	2	102		
4					
5		$2*(1+2)=6$			
6		$102/6=17$			
7		width=17			
8		length=34			
9					

Figure 1: Solving as with pen and paper: solving arithmetically

- (2) Solving as with pen and paper: solving algebraically. Figure 2 describes this method:

	A	B	C	D	E
1					
2			Word Problems		
3					
4			$2x+4x=102$		
5			$6x=102$		
6	width		$x=17$		
7			$x*2=17*2=34$		
8	length		34		
9					

Figure 2: Solving as with pen and paper: solving algebraically

(3) Using the spreadsheets as a calculator, i.e. to compute the values of the unknowns: solving arithmetically. Figures 3(a) and 3(b) describe this method:

	A	B	C	D	E
1					
2	width	length	perimeter		
3	w	2*w	6*w		
4	17	34	102	in our solution we can	
5				change the perimeter	
6				as we wish	
7					
8					
9					

Figure 3(a): using the spreadsheets as a calculator: solving arithmetically

	A	B	C	D	E
1					
2	width	length	perimeter		
3	w	2*w	6*w		
4	=C6/6	=A4*2	102	in our solution we can	
5				change the perimeter as	
6				we wish	
7					
8					
9					

Figure 3(b): using the spreadsheets as a calculator: solving arithmetically – the formulas that describe the calculations

In this case the user worked from the known to the unknown manipulating the known.

(4) Using the spreadsheets as with pen and paper and as a calculator: Solving algebraically. This case is described in figure 4(a) and 4(b) below:

	A	B	C	D	E
1					
2	width	length	perimeter	How we solved	
3	w	2*w	6*w	2*(2w+w)=102	
4	17	34	102	6*a=102	
5				A=102/6	
6					
7					
8					
9					

Figure 4(a): using the spreadsheets as with pen and paper and as a calculator: solving algebraically.

	A	B	C	D	E
1					
2	width	length	perimeter	How we solved	
3	w	2*w	6*w	2*(2w+w)=102	
4	=C6/6	=A4*2	102	6*a=102	
5				A=102/6	
6					
7					
8					
9					

Figure 4(b): using the spreadsheets as with pen and paper and as a calculator: solving algebraically– the formulae

In this case the user worked from the unknown to the known manipulating the unknown.

(5) Building columns with constant difference between the rows. This is done by building columns in the spreadsheets that fit the unknown or the unknowns in the word problem, while taking into account the relations between the unknowns or/and the given. In this word problem the preservice teachers, as themselves or as primary school students, built a column for the width, the length and the perimeter, taking into account the relations between them. Each column is built by dragging the first cell downwards. The first column was built by making the difference between any two consecutive cells in it constant. After building all the columns by dragging, the user looks at the perimeter column to find the cell that contains the given perimeter. Finding this cell, the user looks at the width and length cells beside it, thus he finds the unknowns required in the word problem. Figures 6(a) and 5(b) describe an example of such a method:

	A	B	C
1	perimeter	length	width
2	30	10	5
3	36	12	6
4	42	14	7
5	48	16	8
6	54	18	9
7	60	20	10
8	66	22	11
9	72	24	12
10	78	26	13
11	84	28	14
12	90	30	15
13	96	32	16
14	102	34	17
15	108	36	18
16	114	38	19
17	120	40	20
18	126	42	21

Figure 5(a): building columns by dragging

	A	B	C
1	perimeter	length	width
2	$2*(C2+B2)$	$=2*C2$	5
3	$2*(C3+B3)$	$=2*C3$	6
4	$2*(C4+B4)$	$=2*C4$	7
5	$2*(C5+B5)$	$=2*C5$	8
6	$2*(C6+B6)$	$=2*C6$	9
7	$2*(C7+B7)$	$=2*C7$	10
8	$2*(C8+B8)$	$=2*C8$	11
9	$2*(C9+B9)$	$=2*C9$	12
10	$2*(C10+B10)$	$=2*C10$	13
11	$2*(C11+B11)$	$=2*C11$	14
12	$2*(C12+B12)$	$=2*C12$	15
13	$2*(C13+B13)$	$=2*C13$	16
14	$2*(C14+B14)$	$=2*C14$	17
15	$2*(C15+B15)$	$=2*C15$	18
16	$2*(C16+B16)$	$=2*C16$	19
17	$2*(C17+B17)$	$=2*C17$	20
18	$2*(C18+B18)$	$=2*C18$	21

Figure 5(b): building columns by dragging – formulas

- (6) As in the previous method, i.e. building columns in the spreadsheets that fit the unknown or the unknowns in the word problem, taking into account the different relations between the unknowns or/and the given. The difference here is that building the columns is done without dragging: the preservice teachers build the columns step by step by building at the beginning the first row, looking at the appropriate cell in the ‘given’ column and continuing the columns to modify the result in order to get the given perimeter in the ‘given column’. Figure 6 describes this method:

	A	B	C	D	E
1					
2	width	length	perimeter		
3					
4	5	10	30		
5	7	14	42		
6	10	20	60		
7	12	24	72		
8	15	30	90		
9	17	34	102		

Figure 6: building columns but without dragging

- (7) Building columns without writing equalities in the spreadsheets that describe the relations between the unknowns themselves or/and between them and the given quantities, i.e. computing by heart the length and the perimeter for every substituted width. In this case we have a spreadsheet which looks as in figure 5(a). The difference is that when pointing at any cell we have the same content of the cell and not an algebraic term; i.e. figure 5(b) turns to be figure 5(a). Here too there is no use of the spreadsheets special building potentialities.
- (8) Building just one row, looking at the perimeter cell and comparing it with the given perimeter. If they differ, the user then changes the first cell in the row and starts the process all over again, till he gets a value in the perimeter cell that equals the given perimeter. Figures 7(a) and 7(b) describe this method:

	A	B	C	D	E
1					
2	width	length	perimeter		
3	5	10	30		
4					
5					
6					
7					
8					
9					

Figure 7(a): Solving without columns

	A	B	C	D	E
1					
2	width	length	perimeter		
3	17	$2*A3$	$=2*(A3+B3)$		
4					
5					
6					
7					
8					
9					

Figure 7(b): Solving without columns – The formulae

(9) Using two cells not in a row, as could be seen from figures 8(a) and 8(b):

	A	B	C	D	E
1					
2	width				
3	7				
4			perimeter		
5			42		
6					
7					
8					
9					

Figure 8(a): Using two cells not in a row

	A	B	C	D	E
1					
2	Width				
3	7				
4			perimeter		
5			$2*(A3+2*A3)$		
6					
7					
8					
9					

Figure 8(b): Using two cells not in a row – the formulae

(10) Using the logical function if with a columns' frame, by adding a column that compares the resulting perimeter with the given perimeter and tells if they equal or not. If they equal the cell in the 'if column' returns 'True', otherwise it returns 'False'. Figures 9(a) and 9(b) describes this method:

	A	B	C	D
1	width	length	perimeter	Perimeter=102?
2	5	10	30	False
3	6	12	36	False
4	7	14	42	False
5	8	16	48	False
6	9	18	54	False
7	10	20	60	False
8	11	22	66	False
9	12	24	72	False
10	13	26	78	False
11	14	28	84	False
12	15	30	90	False
13	16	32	96	False
14	17	34	102	True
15	18	36	108	False
16	19	38	114	False
17	20	40	120	False
18	21	42	126	False

Figure 9(a): Using the logical function if with a columns' frame

	A	B	C	D
1	width	length	perimeter	Perimeter=102?
2	5	=2*A2	=2*(A2+B2)	=IF(C2=102, "TRUE","FALSE")
3	6	=2*A3	=2*(A3+B2)	=IF(C3=102, "TRUE","FALSE")
4	7	=2*A4	=2*(A4+B2)	=IF(C4=102, "TRUE","FALSE")
5	8	=2*A5	=2*(A5+B2)	=IF(C5=102, "TRUE","FALSE")
6	9	=2*A6	=2*(A6+B2)	=IF(C6=102, "TRUE","FALSE")
7	10	=2*A7	=2*(A7+B2)	=IF(C7=102, "TRUE","FALSE")
8	11	=2*A8	=2*(A8+B2)	=IF(C8=102, "TRUE","FALSE")
9	12	=2*A9	=2*(A9+B2)	=IF(C9=102, "TRUE","FALSE")
10	13	=2*A10	=2*(A10+B2)	=IF(C10=102, "TRUE","FALSE")
11	14	=2*A11	=2*(A11+B2)	=IF(C11=102, "TRUE","FALSE")
12	15	=2*A12	=2*(A12+B2)	=IF(C12=102, "TRUE","FALSE")
13	16	=2*A13	=2*(A13+B2)	=IF(C13=102, "TRUE","FALSE")
14	17	=2*A14	=2*(A14+B2)	=IF(C14=102, "TRUE","FALSE")
15	18	=2*A15	=2*(A15+B2)	=IF(C15=102, "TRUE","FALSE")
16	19	=2*A16	=2*(A16+B2)	=IF(C16=102, "TRUE","FALSE")
17	20	=2*A17	=2*(A17+B2)	=IF(C17=102, "TRUE","FALSE")
18	21	=2*A18	=2*(A18+B2)	=IF(C18=102, "TRUE","FALSE")

Figure 9(b): Using the logical function if with a columns' frame – the formulae

(11) Using the logical function if with one row frame, as could be seen in Figures 10(a) and 10(b):

	A	B	C	D	E
1					
2	width	length	perimeter	Resulting perimeter = given perimeter?	
3	17	34	102	True	
4					
5					

Figure 10(a): Using the logical function if with one row frame

	A	B	C	D	E
1					
2				Resulting perimeter=	
	width	length	perimeter	given perimeter?	
3	17	2*A3	2*(A3+B3)	=IF(C3=102,	
				"TRUE","FALSE")	
4					
5					

Figure 10(b): Using the logical function if with one row frame – the formulae

- (12) Building just one row, as in the sixth method, but adding a scroll bar though which the user changes the value of the width cell and thus the value of the other cells. Figure 11 describes this method:

	A	B	C	D	E
1	width	length	perimeter		Value of substitution
2	x	2x	2(x+2x)		
3	13	26	87		13
4					
5					

Figure 11: Adding a scroll bar to one row frame

- (13) Building one row with logical if and adding a scroll bar, as could be seen in figure 12.

	A	B	C	D	E	
1			Perimeter	Given	Checking the	▲
	width	length	(result)	perimeter	result	
2	20	40	120	102	False	
3						
4						▼
5						
6						
7						
8						
9						

Figure 12: One row with logical if and with a scroll bar

- (14) Building one row with conditional formatting, so when the cell that contains the calculated perimeter equals the given perimeter the color of the cell changes.
- (15) Building columns with constant difference and adding a combobox that includes the given perimeter quantities. The columns change by choosing a value from the combobox. The columns which change include a cell which contains the required width. This cell has a distinguished color and is adjacent to another cell which included the word 'True'. Tables 13(a) and 13(b) describe this case, where table 13(b) includes but a small part of the actual table. I don't give the whole table for space reasons.

	A	B	C	D	E	F	G
1							102
2							162
3							128
4	perimeter	Length	Width				
	<input type="text" value="162"/>						
5	2	2Y	Y				
6							
7							
8	162		1	2	6	FALSE	
9			2	4	12	FALSE	
10			3	6	18	FALSE	
11			4	8	24	FALSE	
12			5	10	30	FALSE	
13			6	12	36	FALSE	
14			7	14	42	FALSE	
32			25	50	150	FALSE	
33			26	52	156	FALSE	
34			27	54	162	TRUE	
35			28	56	168	FALSE	
36			29	58	174	FALSE	
37			30	60	180	FALSE	
38			31	62	186	FALSE	
39			32	64	192	FALSE	
40			33	66	198	FALSE	
41			34	68	204	FALSE	

Table 13(a): Using a combo box with conditionings

	A	B	C	D	E	F	G
1							102
2							162
3							128
4	perimeter	length	width				
5	162 2	=IF(OR(A6=1,A6=2), "2Y","3Y")	Y				
6							
7							
8	IF(A6=1, 102, IF(A6=2, 162, 128))		1	=IF(OR(\$A\$5=1,\$A\$5=2), 2*C8,3*C8)	=2*(C8 +D8)	=IF(E8=\$A\$8, "TRUE", "FALSE")	
9			2	=IF(OR(\$A\$5=1,\$A\$5=2), 2*C9,3*C9)	=2*(C9 +D9)	=IF(E9=\$A\$8, "TRUE", "FALSE")	

Table 13(b): Using a combo box with conditionings – the formulae

3.2 Solving the second part of ‘The Measurement of a Field’ problem:

Few of the preservice teachers, in both years of the experiment, solved this part using the same frame that they built for part one. Most of them built another frame similar to the one they built for the first part. Those that solved as with pen and paper or used the spreadsheets as a calculator wrote another equation and solved it.

3.3 Solving the third part ‘The Measurement of a Field’ problem:

The majority of the preservice teachers, in both of the experiment, solved this part using a different frame from that they built for the first or second part of the work problem. They did so by changing the algebraic expression for the length of field. Those that solved as with pen and paper or used the spreadsheets as a calculator wrote another equation and solved it.

In table 14 I report the distribution of preservice teachers with respect to the method that they used when they solved the first part of the ‘The Measurement of the field’ problem.

Number and percent of preservice teachers in the year Method used in the spreadsheets environment	2004-2005 N=27		2005-2006 N=34	
	Using the spreadsheets as with pen and paper: solving arithmetically			5
Using the spreadsheets as with pen and paper: solving algebraically	3	11.11%	2	5.88%
Using the spreadsheets as a calculator: solving arithmetically	8	29.63%	3	8.82%
Using the spreadsheets as with pen and paper and as a calculator: solving algebraically	3	11.11%	5	14.7%
Building columns by dragging with constant difference between rows	1	3.7%	4	11.76%
Building columns without dragging (without constant difference between rows)	1	3.7%		
Building columns without writing equalities in the spreadsheets			2	5.88%
Building one row	1	3.7%	2	5.88%
Building two cells not in a row			1	2.94%
Building columns with constant difference with the logical function if	1	3.7%		
Building one row with the logical function if	1	3.7%		
Building one row with a scroll bar	8	29.63%	7	20.59%
Building one row with the logical function if and a scroll bar			3	8.82%
Building columns with a combobox				

Table 14: frequencies (numbers and percents) of methods used to solve the first part of the word problem (solutions by the preservice teachers themselves)

We see from table 14 that some preservice teachers in both years of the study approached the spreadsheets, when working as themselves, as a regular editor for writing mathematical equations. Almost half of the preservice teachers didn't utilize the special features of the spreadsheets environment that distinguish it from other computer software. It's also notable that the mostly used method to utilize the special features of the spreadsheets is the one row frame which is accompanied by a scroll bar.

Method used in the spreadsheets environment	Number and percent of preservice teachers in the year		2004-2005 N=27		2005-2006 N=34	
	Using the spreadsheets as with pen and paper: solving arithmetically					9
Using the spreadsheets as with pen and paper: solving algebraically	2	7.41%				
Using the spreadsheets as a calculator: Solving arithmetically	7	25.93%	3	8.82%		
Using the spreadsheets as with pen and paper and as a calculator: solving algebraically						
Building columns by dragging with constant difference between the rows	8	29.63%	8	23.53%		
Building columns without dragging (without constant difference between the rows)						
Building columns without writing equalities in the spreadsheets						
Building one row	6	22.22%	2	5.88%		
Building two cells not in a row						
Building columns with constant difference with the logical function if	3	11.11%	9	26.47%		
Building one row with the logical function if			1	2.94%		
Building one row with a scroll bar			1	2.94%		
Building one row with the logical function if and a scroll bar			1	2.94%		
Building columns with a combobox	1	3.7%				

Table 15: frequencies (numbers and percents) of methods used to solve the first part of the word problem (solutions as expected by the students)

We see from table 15 that two thirds or almost two thirds of the preservice teachers expect the primary school students to utilize the special features of the spreadsheets environment when they come to solve word problems in this environment. Here we find difference between the utilization of the special features of the spreadsheets by the preservice teachers as them-

selves and as expected by their students. It should be noted that almost half of the preservice teachers, who described the columns frame with the logical function if as a method that primary school students use to solve the word problem, in fact suggested that the teacher prepares this frame for the primary school students. That means that the role of these students is only to fill the width cell below the last row of the row frame and press 'enter', so they get another row which may contain the required perimeter. If it doesn't the students fill the following width cell till they have the required perimeter.

The preservice teachers were requested to try to find a method in which they combine the solutions of the different parts of the word problem. Table 16 shows the frequency of their various approaches to do such combination

Combination used	The year		2004-2005		2005-2006	
			N=27		N=34	
Every part of the word problem was solved alone by the preservice teachers as themselves	17	62.96%	26	76.47%		
The first two parts of the word problem were solved together by the preservice teachers as themselves	10	37.04%	4	11.76%		
The three parts of the word problem were solved together by the preservice teachers as themselves			4	11.76%		
Every part of the word problem was solved alone by the primary school students as expected by the preservice teachers	15	55.56%	27	79.41%		
The first two parts of the word problem were solved together by the primary school students as expected by the preservice teachers	12	44.45%	6	17.65%		
The three parts of the word problem were solved together by the primary school students as expected by the preservice teachers			1	2.94%		

Table 16: frequencies (numbers and percents) of methods used to combine the solutions of the different parts of the 'The Measurement of a Field' problem

We see from table 16 that most of the preservice teachers in both years of the research couldn't build a frame through which they can solve the three parts of the word problem together, but some of them built such a frame for the first two parts.

4. Discussion:

The preservice teachers approached the spreadsheets environment, regarding its building potentialities, in one of three principal ways:

- (1) They considered the spreadsheets as an environment which replaces technically the pen and paper; i.e. as a regular editor.
- (2) They considered the spreadsheets as a calculator that enables them to perform the computations required to solve the word problems.
- (3) Considering the spreadsheets as an environment which has special characteristics which make it different from other environments. This means that they used the special characteristics of the spreadsheets like dragging, functions and controls.

The preservice teachers had mixed approaches too; i.e. they mixed between some of the three approaches mentioned above, as is described in the results.

The preservice teachers approached the spreadsheets differently as themselves and as primary school students, regarding how they utilize the special characteristics of the environment to solve word problems, as if they consider the spreadsheets more appropriate for primary school students to use for solving simple word problems. Though they were required to solve the 'Measurement of a Field' problem in the spreadsheets environment, nearly half of them didn't do that when describing how they themselves would solve, as if they weren't convinced that such a word problem needs a special environment to solve. This consideration changes when we come to how the preservice teachers suggest that primary school students may solve word problems in the spreadsheets environment: almost two thirds of the preservice teachers expect the primary school students to utilize the special characteristics of the spreadsheets environment in order to solve word problems, as if they considered it a tool that helps primary school students to arrange the

known mathematical quantities and the mathematical relations in a special frame in order to arrive at the ‘unknowns’ of the word problem.

Two other issues that are related to utilizing the spreadsheets building potentialities are: (1) the dynamic aspect of the spreadsheets environment and (2) the visual aspect of the spreadsheets environment.

The spreadsheets as dynamic environment:

When the preservice teachers considered the spreadsheets as an environment which replaces technically the pen and paper, they considered it a static environment, but when they considered the spreadsheets as an environment which is different from other environments and utilized its special characteristics, they in fact considered it a dynamic environment that could be programmed to generate dynamic data. The dynamic characteristic is presented in the possibility that changing the content of one cell in the spreadsheet may lead to changing the content of another cell, one row or some columns. This dynamics may vary from one context to another, but it isn't static in any context that utilizes the special characteristics of the spreadsheets environment. Adding a scroll bar increases the dynamics of the computations and thus the spreadsheets environment.

The spreadsheets as visual environment:

The spreadsheets environment has some visual features, of which the preservice teachers used the conditional formatting and the conditional if function. When using the conditional formatting to obtain different colors of a cell or a group of cells, the spreadsheets obtain a visual appearance. In this case it's easy to drag the attention of the user to a cell that has special mathematical property or value. Inserting the logical function if had the same effect in the solutions that the preservice teachers suggested. When using the logical function if to obtain a True/False column or cell, the spreadsheet obtains a visual appearance, as if there is a red/yellow column or cell

where the red color is obtained when the value inside a cell fulfills a specific condition and the yellow color is obtained otherwise.

Regarding how the preservice teachers utilized the algebraic potential of the spreadsheets, i.e. as an environment in which the unknowns, variables and parameters come into being, they had the following approaches:

- (1) They didn't utilize their potential and solved using methods that they use usually outside the spreadsheets environment.
- (2) They considered the spreadsheets as an environment that could generate variables, as when they solved the word problem using one row, one row with a scroll bar, one row with logical if, or one row with a scroll bar and logical if.
- (3) They considered the spreadsheets as an environment that could generate unknowns, as when they solved the word problem using columns. The whole column could be considered unknown that includes what will become the known in one of its cells.
- (4) They considered the spreadsheets as an environment that could generate parameters, as when they solved using the \$ sign, for example in the combo box method.

Having one approach or another, regarding generating variables, unknowns and parameters, depends on one's history of working methods with such mathematical objects in the context given. It also depends on how we look at the tool as appropriate for generating such objects in a given context. Not utilizing the spreadsheets special characteristics for generating variables, unknowns and parameters, when solving word problems, could have been influenced by the preservice teachers' history of solving word problems, which were traditional methods, and by their history of working with the spreadsheets. This history included doing statistical calculations or analysis

in arithmetic contexts, while the history of solving word problems in the primary school, included manipulating numbers that represent the given in the word problem or represent the ratios between the given quantities. This manipulation was done principally horizontally and not vertically as done in the spreadsheets environment. It also involved changing the place of the quantities and not changing the quantities themselves as required in the spreadsheets environment.

Few preservice teachers solved, as themselves, the word problem building a columns frame, while almost quarter of them expected primary school students to solve the word problem by building such a frame. Utilizing the spreadsheets to generate unknowns, in the case of primary school students, could mean that the history of those that suggested columns frame with primary school word problems is a history of working with unknowns represented by shapes and not letters.

Some preservice teachers built one row frame, with or without a scroll bar or/and the 'if' function. This use is the closest to generating a variable. It's also the closest to solving using the 'guessing' method. Almost all the preservice teachers added a scroll bar in their own solutions and not in the expected primary students' solutions. This fact points at different possible things: (1) the preservice teachers think that the primary school students aren't ready yet for the variable notion (2) The preservice teachers think that the primary school students aren't ready yet for building controls in the spreadsheets (3) The preservice teacher wanted to differentiate their solution from the primary students' solutions.

Few preservice teachers used the \$ sign and thus generated parameters. They may have done so because their history of solving word problems doesn't include working with parameters or doesn't include generalizing the solution of the problem to its all parts, in our case three parts.

Regarding the role of the user, the preservice teachers had the following approaches to the role of the user when they come, as themselves or as primary school students, to solve word problems in the spreadsheets environment:

- (1) The user writes starting formulae and fills cells. He improves his substitutions when examining the result of his previous substitutions. He does this till he gets the right answer for the problem. Here the substitution numbers may increase or decrease according to the pattern that the results follow. This could be done while working with one row frame or with columns frame.
- (2) The user writes starting formulae (usually in a couple of consecutive rows) and drags downward till he gets the right answer. Here the substitution numbers always increase or decrease, so the user doesn't judge at every stage if to increase or decrease his substitution.
- (3) The user has a starting frame. He fills a cell which corresponds to some mathematical quantity and presses enter, as when we have a columns frame consisting of two rows.
- (4) The user just clicks twice to get the answer to the word problem, as in the case of the combobox.

The most frequent solution of the preservice teachers themselves is one row frame with a scroll bar, while the most frequent solution that they expect primary school students to have is the columns frame with constant difference (built by dragging). So the preservice teachers found the columns frame more appropriate for primary school students. Using the terminology of Halliday (1985), the columns frame requires less mental processes because, after building the first couple of rows in the frame, all that is required from the student is dragging and looking to see if he gets the right answer in the new row that he gets. If we had had a field with a greater perimeter and the stu-

dent began with a small width, then it would have taken him a longer material process to achieve the width that satisfies the conditions of the word problem. These characteristics of the column frame could be the reason why the preservice teachers suggested that the primary school students would use the columns frame to solve the word problem. On the other hand, writing a starting formula and improving the substitutions require more mental processes because the user not only looks to see if he gets the right answer, but also compares the result that he gets to the known quantity in the word problem and improves his substitution according to the comparing that he did.

Writing a starting formula and improving may be done in more than one way regarding the mathematical quantities involved. For example, for the word problem discussed in this study, the preservice teachers had the following methods: (1) assigning a cell for every mathematical object that the word problem describes: width, length and perimeter (2) assigning cells for part of the mathematical quantities involved: width and perimeter. The first method is more transparent and shows all the mental processes which the user performed, while the second method shows only part of the user's mental processes. The preservice teachers described the second method as a method that they themselves would have, but not the primary school students.

In the third approach described above, the user, who is a student and not a preservice teacher, has a starting frame (without building this frame himself). He fills a cell in the frame which corresponds to some mathematical quantity and presses enter to obtain every time a new row that may contain the right answer. In this method the student performs a material process which is dragging and a mental process which is seeing if he gets the right answer, but not building the first row in the frame lessens the mental processes that the student performs and eliminates an important mental process that the user may perform. Some preservice teachers suggested that the columns frame which the teacher prepares includes the logical function if. This

may result in a student who only looks to see the word 'true', which means that almost all the processes of students in this case are material.

In the fourth approach described above, the user, who is a student and not a preservice teacher, has an almost complete structure, in which he just clicks twice to get the answer to the word problem, as in the case of the combobox. This frame provides an 'answers- included' environment, in which the role of the user is to choose the appropriate perimeter from a set of given numbers. The answer of the word problem is obtained automatically. The choosing process is a behavioral process according to Halliday. Halliday says that behavioral processes are more like doing processes. These processes are needed but not alone.

Regarding combining the solutions of the different parts of the word problem, very few preservice teachers solved the three parts of the word problem using one frame, and few of them solved the first two parts together. What could have combined between the solutions of the three parts is the \$ sign which represents the parameter in the algebraic environment. As mentioned before, the reason for the little use of the \$ sign is the little use that is done with the parameter when solving word problems in the primary school. It was expected that most of the preservice teachers would solve the first two parts of the word problem together because it doesn't involve working with the \$ sign, but few of them did so. This could be explained by the working history of the preservice teachers in the spreadsheets environment. It seems that the preservice teachers had little worked in the spreadsheets environment solving parametric equations or functions or building parametric algebraic expressions.

5. Conclusions and recommendations:

The spreadsheets environment has potentialities for introducing and working with parameters (Dettori et al, 1995), which weren't utilized by the

participants in this research. Learners of mathematics should be aware of these potentialities in order to grasp the notion of the parameter and differentiate between this notion and the notions of the variable and the unknown.

This spreadsheets environment has also potentialities for introducing and working with generalizing. Here we suggest this environment to generalize the solution of word problems. Maybe this introduction isn't appropriate for primary school students, but it could be done in the secondary school or later. Future researches can examine how students work with the spreadsheets environment to generalize the solution of word problems that involve building equations other than linear equations.

References:

1. Abramovich, S. (1995). Technology-Motivated Teaching of Advanced Topics in Discrete Mathematics. *Journal of Computers in Mathematics and Science Education*, 14(3), 1995, 391-418.
2. Abramovich, S. (1996). Exploring prime numbers within a spreadsheet. *International Journal of Mathematics Education in Science and Technology*, 27 (5), 693-699.
3. Abramovich & Nabors, (1996). Exploring Algebraic Word Problems Through Computer-Based Manipulatives and Diverse Technology. Technology and Teacher Education Annual-1996.
4. Abramovich & Nabors, (1997). Spreadsheets as Generators of New Meanings in Middle School Algebra. Computers in Schools, 13(1-2), 13-25.
5. Abramovich, S.; Sugden, S. (2004). Spreadsheet Conditional Formatting: An Untapped Resource for Mathematics Education. *Spreadsheets in Education*, 1 (2), 85-105.

6. Baker, J.; Hvorecky, J. & Sugden, S. (2005). Recursion and Spreadsheets. *Spreadsheets in Education*, 2 (1), 50-72.
7. Baker, J. E. and Sugden, S.J., 2003, Spreadsheets in education - The first 25 years, *Spreadsheets in Education*, v. 1, p. 18-43.
8. Dettori, D.; Garuti, R.; Lemut, E.; Netchitailova, L. (1995). An Analysis of the Relationship between Spreadsheet and Algebra. In Berton and Jaworski (editors), Technology in Mathematics Teaching. Chartwell-Bratt.
9. Drijvers, P.H.M. (2003a). Learning algebra in a computer algebra environment. Design research on the understanding of the concept of parameter. PhD thesis. Utrecht: cd-beta press. <http://www.fi.uu.nl/~pauld/dissertation>.
10. Drijvers, P. (2003b). Algebra on a screen, on paper and in the mind. In J. Fey, A. Cuoco, C. Kieran, L. McMullin & R.M. Zbiek (Eds.), *Computer Algebra Systems in secondary school mathematics education* (pp. 241-267). Reston, VA: National Council of Teachers of Mathematics.
11. Ewusi-Mensah, K., (2003). *Software Development Failures* . Cambridge, MA: Massachusetts Institute of Technology Press.
12. Filloy, E.; Rojano, T. (1989). Solving Equations: the Transition from Arithmetic to Algebra. *For the Learning of Mathematics*, 9(2), 19-25.
13. Guin, D.; Trouche, L.. (1999). "The Complex Process of Converting Tools into Mathematical Instruments: The Case of Calculators." *International Journal of Computers for Mathematical Learning* 3 (3), 195–227.
14. Halliday 1985 : Halliday, M.A.K., *An introduction to functional grammar*. London ; Baltimore, Md., USA : Edward Arnold.

15. Hedberg, J. G. & Perry, N. R. (1985). Human-Computer Interaction and CAI: A review and research prospectus. *Australian Journal of Educational Technology*, 1(1), 12-20.
<http://www.ascilite.org.au/ajet/ajet1/hedberg1.html>
16. Lagrange, J.b. (1999). Complex Calculators in The Classroom: Theoretical and Practical Reflections on Teaching Pre-Calculus. *International Journal of Computers for Mathematical Learning* 4 (1), 51–81.
17. Neuwirth, E. (1995). Spreadsheet structures as a model for proving combinatorial identities. *Journal of Computers in Mathematics and Science Education*, 14(3), 419-434.
18. Rojano, T., (1996). Developing Algebraic Aspects of Problem Solving Within a Spreadsheet Environment. In Bednarz, Kieran, and Lee (editors), Approaches to Algebra, Perspectives for Research and Teaching. Kluwer Academic Publishers.
19. Smith, R. S., (2005). Spreadsheets in the Mathematics Classroom. *Proceedings of the 1st KAIST International Symposium on Enhancing University Mathematics Teaching*. Korea.
20. Soper, J. B. & Lee, M. P., (1994). Computer spreadsheets for numerical analysis. *International Journal of Mathematics, Education, Science and Technology*, 25 (2), 245-253.
21. Sutherland, R.; Rojano, T., (1993). A Spreadsheet Approach to Solving Algebra Problems. *Journal of Mathematical Behaviour*, 12, 353-383.
22. Tall, D., (1993). The Transition from Arithmetic to Algebra: Number Patterns or Proceptual Programming? *New Directions in Algebra Education*, Queensland University of Technology, Brisbane, 213-231. Available on the Internet at:
http://www.warwick.ac.uk/staff/David.Tall/List_of_papers.htm.